9. É. K. Kalinin, I. I. Berlin, V. V. Kostyuk, and Yu. S. Kochelaev, Heat Transfer during Film Boiling in Components of Power Apparatus [in Russian], VINITI, Moscow (1972).
10. L. B. Bulanova and V. G. Pron'ko, in: Transactions of the All-Union Scientific-Research Institute of Cryogenic Machines [in Russian], Vol. 14 (1975), p. 307.
11. C. Massot and F. Frani, AIChE J., 12, No. 3 (1966).
12. A. V. Lykov, Theory of Heat Conduction [in Russian], Vysshaya Shkola, Moscow (1967).

HEAT TRANSFER FOR A LIQUID BOILING IN A BED OF GRANULAR MATERIAL

G. F. Smirnov

UDC 536.423.1:541.182

An approximate analytical model is given for heat transfer during boiling in a bed of granular material, and this is used in a generalization from the published data.

Boiling in granular beds is a promising technique for the design of heat-transfer systems [1, 2] and other types of process plant [3-6]. This requires a better understanding of the physical processes in such beds. There were marked differences between the conditions of [1, 2] and of [3, 4, 7], as Fig. 1 shows, since $H/b \ll 1$ in [1, 2] for $\rho_g/\rho' \gg 1$, whereas $H/b \gg 1$ for $\rho_g/\rho' = 1.5$-3 in [3, 4, 7]. Therefore, one assumes that there was gravitational pressure of the particles on the wall in [1, 2]. On the other hand, in [3, 4, 7] the particles near the wall were largely relieved from the pressure of the overlying layers, $H/b \gg 1$, and so they were probably displaced from the wall. In [7], this displacement was detected photographically. If one assumes that the main heat-transport mechanism is determined by the processes at the wall for boiling in a bed, then there should be an analogy with a process in a horizontal slot having an equivalent width which is proportional to the particle size. Comparison of [3, 4, 7] with [10, 11] confirms that the behavior of the heat-transfer coefficients is much the same in both cases. On the other hand, boiling in a bed differs from that in a horizontal slot in that the gas–liquid mixture migrates in much the same way as in fluidization. The smaller the particles and the shallower the bed, or the higher the thermal loading, the greater the influence of fluidization. The particles can be completely suspended by a thermal fluidization mechanism, and then the main heat-transfer mechanism would be as for a free volume of liquid. The particles have an effect via the effective thermophysical characteristics of the liquid–particle medium (viscosity μ_e, thermal conductivity λ_e, specific heat c_e, etc.). Therefore, standard equations for heat transfer in boiling can be used with the effective constants and the semiempirical relationships for horizontal slots [10, 11] to construct an approximate model for boiling in a displaced granular bed. It is then very important to define a criterion that determines the contribution from any particular mechanism. This is possible if we compare the characteristic boiling rate $u = q/r\rho''$ with a characteristic of the fluidization, namely, the critical fluidization speed $u_{cr} = C_{cr}gd^2 \cdot (\rho_g - \rho')/\mu'$ [12], where C_{cr} is a numerical constant dependent on the mode of flow around a suspended particle. Therefore, we have for the total heat flux in this granular bed that

$$q = q_1(1-\psi) + q_0\psi, \tag{1}$$

where q_1 is the heat flux transported in the process analogous to boiling in a horizontal slot and q_0 is the heat flux transported by the nominal process analogous to that in a free volume of liquid with the effective thermophysical parameters, while ψ is an interpolation factor, which is the ratio u/u_{cr}, which can be put in the following form on passing to the limit:

$$\psi = \frac{u}{u_{cr} + u}. \tag{2}$$

Further, q_0 is dependent on the working conditions, the physical and geometrical parameters, and so on [13-15, et al.]. A semiempirical relationship for q_1 has been derived from the experimental data of [10, 11] by means of a physical model for boiling in a horizontal slot. The basic features of this model are as follows.

Odessa Refrigeration-Technology Institute. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 34, No. 5, pp. 792-798, May, 1978. Original article submitted January 13, 1977.

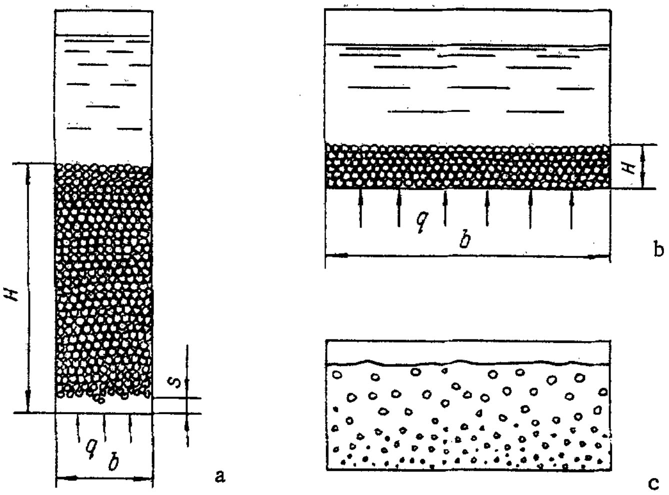

Fig. 1. Basic cases of boiling in a granular bed: a) $H \gg b$, partial or complete displacement of the bed, $s \sim d$, $u \ll u_{cr}$; b) $H < b$, $\rho_g \gg \rho'$; $u \ll u_{cr}$, gravitational compression of the bed [1, 2]; c) $u \gg u_{cr}$, bed completely fluidized.

1. A combined heat-transfer mechanism applies; heat is brought up to the phase interface via a microscopic layer of liquid when the vapor is attached to the wall. When the liquid is in contact with the wall, we get contact or convective heat transfer.

2. If the horizontal slot is narrow, we get thermal conduction through a thin layer of liquid of some initial thickness δ, in which the specific thermal resistance is

$$R_M = \delta/\lambda'. \tag{3}$$

3. There is a relationship between the initial thickness of this microlayer and the local hydrodynamic characteristics (speed $\dot{R}$ of the phase interface and pressure gradient dP/dR), which is much the same as the relationship between the thickness δ of a lubrication layer for the same $\dot{R}$ and dP/dR [16]:

$$\delta \simeq C_M \sqrt{\mu' \dot{R} / \left|\frac{dP}{dR}\right|}. \tag{4}$$

4. The hydrodynamics of these thin layers may be considered via the motion of the phase interface in a system of growing vapor bubbles whose density is governed by the laws applicable for boiling in a free volume [14]:

$$n_F \sim \left(\frac{r\rho''\Delta T}{\sigma T_s}\right)^m, \tag{5}$$

where m lies in the range 1-2.

5. The mean excess vapor pressure P within a vapor bubble is governed mainly by the conditions for displacement of the liquid from the slot:

$$P - P_s \sim C_p \rho' w^2. \tag{6}$$

6. The mean speed w of displacement of the liquid from the slot of width b is given by the equation of continuity as

$$w = n_F b \pi R \dot{R}. \tag{7}$$

7. A simple law applies for the rate of growth $\dot{R}$ of the bubbles in a gap of width s; an energy scheme on the basis that $q \sim$ const gives

$$\dot{R} \simeq \frac{C_R R q}{r\rho'' s}. \tag{8}$$

Joint use of (3)-(8) with $s \simeq C_d d$ gives

$$\frac{\delta}{\lambda'} = R_M = \frac{\text{const}}{b\lambda'} \sqrt{\frac{r\rho'' d\nu'}{q n_F}}. \tag{9}$$

While it is possible to assume that m = 2 [14] for a horizontal slot, as for a free liquid, it is likely that m < 2 for a displaced bed of the above type; in order not to alter the dimensional coefficient in (5) and (9), it is best to put (5) as

$$n_F L^2 \sim \left(\frac{r\rho''\Delta T}{\sigma T_s} L\right)^m \tag{10}$$

Since L is a function of the properties and the pressure only for a free liquid, we can use any standard formula for L, e.g.,

$$L^2 \sim \sigma/(\rho' - \rho'')\, g. \tag{11}$$

Then (9)-(11) gives us that

$$q_1 = \text{const}\,(b\lambda')^2 \left[\frac{r\rho''}{\sqrt{\sigma(\rho' - \rho'')\, g}\, T_s}\right]^m \frac{g(\rho' - \rho'')\,\Delta T^{2+m}}{\sigma r\rho'' d\nu'}. \tag{12}$$

If the heat-transfer conditions in a granular layer are substantially different from those in a free liquid, i.e., $q \gg q_0$ and $q_1(1-\psi) \gg q_0\psi$, we can put that

$$q \simeq q_1(1-\psi). \tag{13}$$

We then use (2), (12), and the values for u and u_{cr} to get

$$q = \text{const}\,(b\lambda')^3 \left[\frac{r\rho''}{\sqrt{\sigma(\rho' - \rho'')g}\, T_s}\right]^m \frac{g(\rho' - \rho'')\,\Delta T^{2+m}}{\sigma r\rho'' d\nu'} \frac{1}{1 + C_q \dfrac{q\nu'\rho'}{r\rho'' g d^2(\rho_g - \rho')}}. \tag{14}$$

We solve (14) for q to get an equation that can be used in fitting to the experimental data; if m = 1,

$$Z = C_1(\sqrt{1 + C_2 y^3} - 1), \tag{15}$$

where

$$Z = \frac{q\nu'}{r\rho'' g d^2} \cdot \frac{\rho'}{\rho_g - \rho'};$$
$$y = \left[\frac{1}{\sigma T_s} \cdot \frac{\rho'}{\rho_g - \rho'} \cdot \frac{(b\lambda')^2}{g r\rho'' d^3} \sqrt{\frac{(\rho' - \rho'')g}{\sigma}}\right]^{1/3} \Delta T; \tag{16}$$

for m = 2

$$Z = C_1' \left[\sqrt{1 + C_2'(y')^4} - 1\right]; \tag{17}$$

$$y' = \left[\frac{1}{\sigma T_s} \cdot \frac{\rho'}{\rho_g - \rho'} \cdot \frac{(b\lambda')^2}{r\rho'' g d^3} \sqrt{\frac{(\rho' - \rho'')g}{\sigma}}\right]^{1/4} \Delta T. \tag{18}$$

Figure 2 shows measurements [3, 4] for boiling in a granular bed in the form of (16), as well as the result from (15); the model not only fits the available evidence, but also explains the effects of various factors. For example, (15) implies that there is a value of d corresponding to maximum heat flux for a given ΔT, a given pressure, and a given liquid, which agrees with experiment [17]. Also, (15) implies that n is inversely related to ΔT in $q = C_g \Delta T^n$; for instance, n is 3-4 if ΔT is small, but it falls to 1.5 at high ΔT. This is also in agreement with experiment [4]. The model also incorporates correctly other features such as the degeneration of the accelerated heat transfer when the particles become small or the heat flux increases, the independence of the heat-transfer rate of the bed depth for H > (2-3)b, and the reduced gain from using the bed when the saturation pressure falls to P_s < 0.1 bar. The laws applicable to boiling in a granular bed at the outside of a horizontal tube are thus roughly the same as for boiling on a plate of width approximately equal to the outside diameter of the tube. This is also confirmed by experiment [4]. A bundle of tubes immersed in a granular bed will also give the same results as for a single tube if the heat loading is small. The circulation becomes accentuated as q increases, on account of interaction between the tubes in a given vertical series. The circulation caused by the bundle of tubes (i.e., N tubes in a single vertical series) will accentuate the fluidization, and this can be incorporated by means of a factor of $(1-\psi)'$ type, but with the difference that the characteristic velocity is replaced by the circulation velocity w_c. To a first approximation, $w_c \sim N(q/r\rho'')$, and

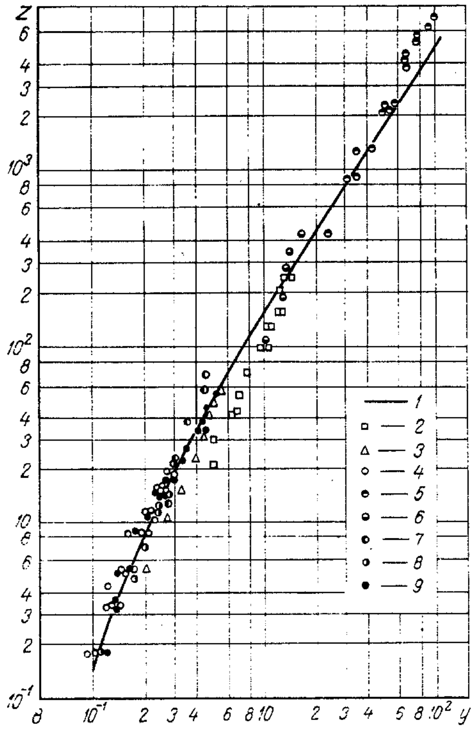

Fig. 2. Curve fitted to observations derived from analytical investigation: 1) from (15) for C_1 = 6.5; C_2 = 0.37 ($z = 6.5\sqrt{1 + 0.37y^3} - 1$); 2-4) [3], H_2O, P_s = 1 bar; 2) d = $0.63 \cdot 10^{-3}$; ρ_g = 2700 kg/m³; 3) d = $1.33 \cdot 10^{-3}$; 4) d = $2.0 \cdot 10^{-3}$; 5-9) [4], H_2O: 5) d = $0.1 \cdot 10^{-3}$; $\rho_g = 4 \cdot 10^3$; P_s = 1 bar; 6) respectively, $2.3 \cdot 10^{-3}$; $2.7 \cdot 10^3$; 0.035 bar; 7) $2.3 \cdot 10^{-3}$; $2.7 \cdot 10^3$; 0.2 bar; 8) $2.3 \cdot 10^{-3}$; $2.7 \cdot 10^3$; 0.2 bar; 9) $2.3 \cdot 10^{-3}$; $2.7 \cdot 10^3$; 1 bar.

$$z = \frac{qv'}{r\rho'' gd^2} \cdot \frac{\rho'}{\rho_g - \rho'}; \quad y = \left[\frac{1}{\sigma T_s} \cdot \frac{\rho'}{\rho_g - \rho'} \cdot \frac{(b\lambda')^2}{r\rho'' gd^3} \sqrt{\frac{\rho' - \rho''}{\sigma} g}\right]^{1/3} \Delta T$$

$$(1 - \psi') \cong \frac{1}{1 + N \dfrac{qv'}{r\rho'' gd^2} \cdot \dfrac{\rho'}{\rho_g - \rho'} C_N}. \tag{19}$$

Then (14) gives us for a set of tubes that

$$q = \mathrm{const}\,(b\lambda')^2 \left[\frac{r\rho''}{\sqrt{\sigma(\rho' - \rho'') g T_s}}\right]^m \frac{(\rho' - \rho'') g}{\sigma} \cdot \frac{\Delta T^{2+m}}{r\rho'' dv'} \times$$
$$\times \frac{1}{1 + C_q \dfrac{qv'\rho'}{r\rho'' gd^2(\rho_g - \rho')}} \cdot \frac{1}{1 + C_N N \dfrac{qv'\rho'}{r\rho'' gd^2(\rho_g - \rho')}}. \tag{20}$$

We see from (20) that $q = f(\Delta T)$ tends to the form $q \propto \Delta T$ as q increases, which agrees with experiment [4].

This scheme thus provides a sound basis for fitting a curve to the observations and gives a quantitative explanation of various laws that have been observed.

A difference from [17] is that we have incorporated the width of the heater b, the density ρ_s of the solid phase, and so on, while fitting to the published data [3, 4], in which a functional relationship was derived also; however, our relationship contains only two empirical constants. A full check on our scheme will, of course, require a volume of experimental data much larger than that of [3, 4, 17].

NOTATION

q, heat flux; u, characteristic velocity; ρ_g, granule density; r, ρ'', ρ', ν', μ', λ', σ, heat of evaporation, vapor and liquid phase densities, kinematic and dynamic viscosities, thermal conductivity of liquid, and surface tension, respectively; P, P_s, T_s, pressure in a bubble, pressure and saturation temperature in liquid; R, $\dot{R}$, dP/dR, radius, velocity of phase interface, and pressure gradient for a bubble in horizontal slot formed during bed separation; δ, R_M, initial thickness of microbed and thermal resistance; ΔT, mean wall—liquid temperature difference; s, width of slot formed during bed separation; b, plate width and tube diameter; w, mean velocity of liquid flowing out of slot during bubble growth; n_F, concentration of evaporation sites; g, acceleration of gravity; L, characteristic dimension for vapor phase during boiling; N, number of tubes in a vertical row; H, bed height; w_c, mean velocity of circulation due to boiling on a bundle immersed in bed; C_M C_p, C_R, C_q, C_1, C_2, C_1', C_2', C_g, C_N, numerical coefficients. Indices: s, vapor—liquid equilibrium; M, microbed.

LITERATURE CITED

1. D. C. Ferell and W. C. Johnson, in: Heat Pipes [Russian translation], Mir, Moscow (1972), p. 9.
2. D. C. Ferell and D. Olliwicz, in: Heat Pipes [Russian translation], Mir, Moscow (1972), p. 118.
3. M. I. Berman and Z. R. Gorbis, Teploénergetika, No. 11 (1973).
4. Z. R. Gorbis and M. I. Berman, Inzh.-Fiz. Zh., 27, No. 3 (1974).
5. T. A. Kolach, R. Kh. Sharipov, and V. V. Yagov, Inzh.-Fiz. Zh., 14, No. 6 (1968).
6. G. M. Pludovskaya, Tr. Tsentr. Nauchno-Issled. Kotloturb. Inst., No. 101 (1970).
7. Z. R. Gorbis, G. F. Smirnov, and G. A. Savchenkov, Inzh.-Fiz. Zh., 24, No. 3 (1974).
8. J. C. Corman and G. E. Walmet, "Vaporization from capillary wick structures," Pap. ASME, NHT-35 (1971).
9. A. Abhat and R. A. Seban, in: Heat Transfer, Proceedings of the American Society of Mechanical Engineers, No. 3 (1974), p. 74.
10. A. L. Koba and G. F. Smirnov, Inzh.-Fiz. Zh., 27, No. 4 (1974).
11. G. F. Smirnov, A. L. Koba, B. A. Afanas'ev, and V. V. Zrodnikov, in: Heat and Mass Transfer [in Russian], Vol. 3, Minsk (1976), p. 193.
12. M. É. Azrov and O. M. Todes, Hydraulic and Thermal Working Conditions of Equipment Containing Stationary and Fluidized Granular Beds [in Russian], Khimiya, Leningrad (1968).
13. S. S. Kutateladze, Principles of Heat Transfer [in Russian], Sibirsk. Otd. Akad. Nauk SSSR, Nauka, Novosibirsk (1970).
14. D. A. Labuntsov, Izv. Akad. Nauk SSSR, Énerget. Transport, No. 1 (1963).
15. V. M. Borishanskii and A. I. Kozyrev, Inzh.-Fiz. Zh., 2, No. 12 (1962).
16. G. F. Smirnov, Inzh.-Fiz. Zh., 28, No. 3 (1975).
17. Z. R. Gorbis and M. I. Berman, Heat and Mass Transfer Conference-5 [in Russian], Vol. 3, Part 1 (1976).